

Formula Sheet

Maxwell's Equations:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0} \quad (\text{Gauss's law for } \vec{E})$$

$$\oint \vec{B} \cdot d\vec{A} = 0 \quad (\text{Gauss's law for } \vec{B})$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(i_C + \epsilon_0 \frac{d\Phi_E}{dt} \right)_{\text{encl}} \quad (\text{Ampere's law})$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \quad (\text{Faraday's law})$$

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y(x,t)}{\partial t^2} \quad (\text{wave equation})$$

$$v = \sqrt{\frac{F}{\mu}} \quad (\text{speed of a transverse wave on a string})$$

$$P_{\text{av}} = \frac{1}{2} \sqrt{\mu F} \omega^2 A^2 \quad (\text{average power, sinusoidal wave on a string})$$

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} \quad (\text{inverse-square law for intensity})$$

$$y(x,t) = y_1(x,t) + y_2(x,t) \quad (\text{principle of superposition})$$

Lorentz Force Law:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

Standing Waves:

$$f_n = n \frac{v}{2L} = nf_1 \quad (n=1,2,3,\dots)$$

Doppler effect:

$$f_L = \frac{v + v_L}{v + v_S} f_S$$

Mechanical Waves:

$$v = \lambda f \quad (\text{periodic wave})$$

$$y(x,t) = A \cos \left[\omega \left(\frac{x}{v} - t \right) \right] = A \cos \left[2\pi f \left(\frac{x}{v} - t \right) \right] \quad (\text{sinusoidal wave})$$

$$y(x,t) = A \cos \left[2\pi \left(\frac{x}{\lambda} - \frac{t}{T} \right) \right] \quad (\text{sinusoidal wave moving})$$

$$y(x,t) = A \cos(kx - \omega t) \quad (\text{sinusoidal wave moving in})$$

Electric Field:

$$F = \frac{1}{4\pi\epsilon_0} \frac{|q_1 q_2|}{r^2} \quad (\text{Coulomb's law: force between two point charges})$$

$$\vec{E} = \frac{\vec{F}_0}{q_0} \quad (\text{definition of electric field as electric force per unit charge})$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} \quad (\text{electric field of a point charge})$$

$$\tau = pE \sin \phi \quad (\text{magnitude of the torque on an electric dipole})$$

$$\vec{\tau} = \vec{p} \times \vec{E} \quad (\text{torque on an electric dipole, in vector form})$$

$$U = -\vec{p} \cdot \vec{E} \quad (\text{potential energy for a dipole in an electric field})$$

Electric Potential:

$$W_{a \rightarrow b} = U_a - U_b = -(U_b - U_a) = -\Delta U \quad (\text{work done by a conservative force})$$

$$U = \frac{1}{4\pi\epsilon_0} \frac{qq_0}{r} \quad (\text{electric potential energy of two point charges } q \text{ and } q_0)$$

$$U = \frac{q_0}{4\pi\epsilon_0} \left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \dots \right) = \frac{q_0}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i} \quad (\text{point charge } q_0 \text{ and collection of charges } q_i)$$

$$V = \frac{U}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{q}{r} \quad (\text{potential due to a point charge})$$

$$V = \frac{U}{q_0} = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i} \quad (\text{potential due to a collection of point charges})$$

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r} \quad (\text{potential due to a continuous distribution of charge})$$

$$V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l} = \int_a^b E \cos \phi \, dl \quad (\text{potential difference as an integral of electric field})$$

$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z} \quad (\text{components of } \vec{E} \text{ in terms of } V)$$

$$\vec{E} = -\left(\hat{i} \frac{\partial V}{\partial x} + \hat{j} \frac{\partial V}{\partial y} + \hat{k} \frac{\partial V}{\partial z} \right) \quad (\vec{E} \text{ in terms of } V)$$

Capacitance:

$$C = \frac{Q}{V_{ab}} \quad (\text{definition of capacitance})$$

$$C = \frac{Q}{V_{ab}} = \epsilon_0 \frac{A}{d} \quad (\text{capacitance of a parallel-plate capacitor in vacuum})$$

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots \quad (\text{capacitors in series})$$

$$C_{\text{eq}} = C_1 + C_2 + C_3 + \dots \quad (\text{capacitors in parallel})$$

$$U = \frac{Q^2}{2C} = \frac{1}{2} CV^2 = \frac{1}{2} QV \quad (\text{potential energy stored in a capacitor})$$

$$u = \frac{1}{2} \epsilon_0 E^2 \quad (\text{electric energy density in a vacuum})$$

Current, Resistance, and Ohm's Law

$$I = \frac{dQ}{dt} = n|q|v_d A \quad (\text{general expression for current})$$

$$\vec{J} = nq\vec{v}_d \quad (\text{vector current density})$$

$$\rho = \frac{E}{J} \quad (\text{definition of resistivity})$$

$$\rho(T) = \rho_0[1 + \alpha(T - T_0)] \quad (\text{temperature dependence of resistivity})$$

$$R = \frac{\rho L}{A} \quad (\text{relationship between resistance and resistivity})$$

$$V = IR \quad (\text{relationship among voltage, current, and resistance})$$

$$V_{ab} = \mathcal{E} - Ir \quad (\text{terminal voltage, source with internal resistance})$$

$$P = V_{ab}I \quad (\text{rate at which energy is delivered to or extracted from a circuit element})$$

$$P = V_{ab}I = I^2R = \frac{V_{ab}^2}{R} \quad (\text{power delivered to a resistor})$$

$$R_{\text{eq}} = R_1 + R_2 + R_3 + \dots \quad (\text{resistors in series})$$

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots \quad (\text{resistors in parallel})$$

$$\sum I = 0 \quad (\text{junction rule, valid at any junction})$$

$$\sum V = 0 \quad (\text{loop rule, valid for any closed loop})$$

$$q = C\mathcal{E}(1 - e^{-t/RC}) = Q_f(1 - e^{-t/RC}) \quad (R-C \text{ circuit, charging capacitor})$$

$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R} e^{-t/RC} = I_0 e^{-t/RC} \quad (R-C \text{ circuit, charging capacitor current})$$

$$q = Q_0 e^{-t/RC} \quad (R-C \text{ circuit, discharging capacitor})$$

$$i = \frac{dq}{dt} = -\frac{Q_0}{RC} e^{-t/RC} = -I_0 e^{-t/RC} \quad (R-C \text{ circuit, discharging capacitor current})$$

Magnetic Force and Magnetic Flux:

$$\vec{F} = q\vec{v} \times \vec{B} \quad (\text{magnetic force on a moving charged particle})$$

$$\Phi_B = \int B_{\perp} dA = \int B \cos \phi dA = \int \vec{B} \cdot d\vec{A} \quad (\text{magnetic flux through a surface})$$

$$\oint \vec{B} \cdot d\vec{A} = 0 \quad (\text{magnetic flux through any closed surface})$$

$$R = \frac{mv}{|q|B} \quad (\text{radius of a circular orbit in a magnetic field})$$

$$\vec{F} = I\vec{l} \times \vec{B} \quad (\text{magnetic force on a straight wire segment})$$

$$d\vec{F} = Id\vec{l} \times \vec{B} \quad (\text{magnetic force on an infinitesimal wire section})$$

$$\tau = IBAsin\phi \quad (\text{magnitude of torque on a current loop})$$

$$\vec{\tau} = \vec{\mu} \times \vec{B} \quad (\text{vector torque on a current loop})$$

$$U = -\vec{\mu} \cdot \vec{B} = -\mu B \cos \phi \quad (\text{potential energy for a magnetic dipole})$$

Magnetic Field:

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2} \quad (\text{magnetic field of a point charge with constant velocity})$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{l} \times \hat{r}}{r^2} \quad (\text{magnetic field of a current element})$$

$$B = \frac{\mu_0 I}{2\pi r} \quad (\text{near a long, straight, current-carrying conductor})$$

$$\frac{F}{L} = \frac{\mu_0 II'}{2\pi r} \quad (\text{two long, parallel, current-carrying conductors})$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$$

Inductance and Displacement Current:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \quad (\text{Faraday's law of induction})$$

$$\mathcal{E} = vBL \quad (\text{motional emf; length and velocity perpendicular to } \vec{B})$$

$$\mathcal{E} = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l} \quad (\text{motional emf; closed conducting loop})$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \quad (\text{stationary integration path})$$

$$i_D = \epsilon_0 \frac{d\Phi_E}{dt} \quad (\text{displacement current})$$

$$M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{N_1 \Phi_{B1}}{i_2} \quad (\text{mutual inductance})$$

$$L = \frac{N\Phi_B}{i} \quad (\text{self-inductance})$$

$$\mathcal{E} = -L \frac{di}{dt} \quad (\text{self-induced emf})$$

$$U = L \int_0^1 i di = \frac{1}{2} LI^2 \quad (\text{energy stored in an inductor})$$

$$u = \frac{B^2}{2\mu_0} \quad (\text{magnetic energy density in vacuum})$$

$$u = \frac{B^2}{2\mu} \quad (\text{magnetic energy density in a material})$$

$$\tau = \frac{L}{R} \quad (\text{time constant for an } R\text{-}L \text{ circuit})$$

$$\omega = \frac{1}{\sqrt{LC}} \quad (\text{angular frequency of oscillation in an } L\text{-}C \text{ circuit})$$

$$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \quad (\text{underdamped } L\text{-}R\text{-}C \text{ series circuit})$$

Electromagnetic Waves:

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \quad (\text{speed of electromagnetic waves in vacuum})$$

$$\vec{E}(x,t) = \hat{j} E_{\max} \cos(kx - \omega t)$$

$$\vec{B}(x,t) = \hat{k} B_{\max} \cos(kx - \omega t)$$

$$E_{\max} = c B_{\max} \quad (\text{electromagnetic wave in vacuum})$$

$$E_y(x,t) = E_{\max} \cos(kx + \omega t) \quad B_z(x,t) = -B_{\max} \cos(kx + \omega t)$$

(sinusoidal electromagnetic plane wave, propagating in $-x$ -direction)

$$c = \frac{1}{\sqrt{\epsilon \mu}} = \frac{1}{\sqrt{K K_m}} \frac{1}{\sqrt{\epsilon_0 \mu_0}} = \frac{c}{\sqrt{K K_m}} \quad (\text{speed of electromagnetic waves})$$

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \quad (\text{Poynting vector in vacuum})$$

$$n = \frac{c}{v} \quad (\text{index of refraction})$$

$$\theta_r = \theta_a \quad (\text{law of reflection})$$

$$n_a \sin \theta_a = n_b \sin \theta_b \quad (\text{law of refraction})$$

Volume and Area Formulae:

Circle:

$$\text{Area:} \quad \pi r^2$$

$$\text{Circumference:} \quad 2\pi r$$

Sphere:

$$\text{Area:} \quad 4\pi r^2$$

$$\text{Volume:} \quad \frac{4}{3} \pi r^3$$

Cylinder:

$$\text{Area:} \quad 2\pi r l$$

$$\text{Volume:} \quad \pi r^2 l$$

Fundamental Constants:

$$c = 2.9979 \cdot 10^8 \text{ m/s}$$

$$k = 9.0 \cdot 10^9 \text{ Nm}^2/\text{C}^2$$

$$\epsilon_0 = 4\pi/k = 8.85 \cdot 10^{-12} \text{ C}^2/\text{Nm}^2$$

$$\mu_0 = 4\pi \cdot 10^{-7} \text{ TA/m}$$

$$q_e = 1.602 \cdot 10^{-19} \text{ C}$$

$$m_e = 9.11 \cdot 10^{-31} \text{ kg}$$